Bayesian Networks and Influence Diagrams: A Guide to Construction and Analysis

Uffe B. Kjærulff and Anders L. Madsen

Answers to Exercises

August 3, 2009

Answer to Exercise 11.1: See table below

(a)

	B	L	T
H(X)	0.6881	0.213	0.0578

(b)

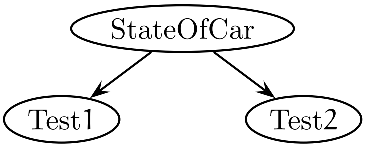
	A	D	S	X
I(B, Y)	0	0.2506	0.0462	0.0008
I(L, Y)	0	0.0176	0.0224	0.1281
I(T, Y)	0.0004	0.0028	0	0.0219

(c)

	A	D	S	X
I(B, Y)	≈ 0	0	0	0.0275
I(L, Y)	≈ 0	0	0	0.2819
I(T, Y)	0.0006	0	0	0.0233

Answer to Exercise 11.2:

(a)



The variable StateOfCar (SOC) is associated with $P(SOC) = (0.7, 0.3)$ and for the other two variables we have:

		Test1		
		¬d	d	inc.
SOC	¬d	0.8	0.05	0.15
	d	0.05	0.75	0.2

		Test2	
		¬d	d
SOC	¬d	0.8	0.2
	d	0.25	0.75

(b) We have:

$$P(SOC \mid Test1) = \frac{P(SOC, Test1)}{P(Test1)} = \frac{P(Test1 \mid SOC)P(SOC)}{\sum_{SOC} P(Test1 \mid SOC)P(SOC)}$$

Using the tables we get (for $P(SOC, Test1)$):

		Test1		
		¬d	d	inc.
SOC	¬d	0.56	0.035	0.105
	d	0.015	0.225	0.06

$P(SOC, Test1)$

Hence, $P(Test1) = (0.575, 0.26, 0.165)$ and finally:

		Test1		
		¬d	d	inc.
SOC	¬d	0.974	0.135	0.636
	d	0.026	0.865	0.364

$P(SOC \mid Test1)$

(c)

	$Test_1$	$Test_2$
I(SOC, Y)	0.3305	0.1373

Answer to Exercise 11.3:

(a) Through construction of the influence diagram in HUGIN and performing inference without evidence (no information is available prior to taking decision D_1), we get the expected utilities 17.7 and 19.0 of performing actions d_{11} and d_{12}, respectively. Thus, we should first perform action d_{12}. From the influence diagram it appears that the value of variable T is known prior to taking decision D_2. Thus, by instantiating D_1 to d_{12} and T to t_1 and t_2 we get, respectively,

$$\text{EU}(D_2 \mid D_1 = d_{12}, T = t_1) = (18.3, 15.2)$$

and

$$\text{EU}(D_2 \mid D_1 = d_{12}, T = t_2) = (19.6, 10.4).$$

Thus, since the expected utility of decision d_{21} is greater than the expected utility of decision d_{22} no matter the value of T, the solution to the decision problem is given by the sequence (d_{12}, d_{21}) of decisions.

(b) The information on variable T makes no difference, as the expected utility of decision d_{21} is greater than the expected utility of decision d_{22} no matter the value of T.

(c) With these modifications we get

$$\text{EU}(D_1) = (50.8, 51.3),$$

$$\text{EU}(D_2 \mid D_1 = d_{12}, T = t_1) = (57.8, 32.7),$$

and

$$\text{EU}(D_2 \mid D_1 = d_{12}, T = t_2) = (40.2, 46.8).$$

That is, now the information on variable T makes a difference, and we get solutions (d_{12}, d_{21}) if $T = t_1$ and (d_{12}, d_{22}) if $T = t_2$.

(d) Under the original quantification $\text{VOI}(T) = 0$ as the optimal decision does not depend on the observation made on T. Under the revised quantification $\text{VOI}(T) = 10.224$ as the optimal decision depends on the observation made on T.

Answer to Exercise 11.4:

(a) First we calculate $P(\text{Flu}) = (0.01; 0.99)$, $P(\text{Fever}) = (0.10607; 0.89393)$, and:

		Flu	
		yes	no
Fever	yes	0.08532	0.91468
	no	0.00106	0.99894

$P(\text{Flu} \mid \text{Fever})$

With $U(\text{Buy}, \text{Flu})$:

		Flu	
		yes	no
Buy	yes	1850	1850
	no	0	2000

we get:

$$\text{EU}(\text{Buy}) = (1850, 1980) \quad (1)$$

hence, $V(P(\text{Flu})) = 1980$.

Now $\mathsf{EU}(\mathsf{buy} \mid \mathsf{fever}) =$

		buy: yes	buy: no
fever	yes	1850	1829.36
	no	1850	2000

thus, $\mathsf{V}(\mathsf{P}(\mathsf{flu} \mid \mathsf{fever})) = (1850, 1997.88)$ and in particular:

$$\begin{aligned} \mathsf{EV}(\mathsf{fever}) &= 1982.19; \\ \mathsf{EB} &= 1982.19 - 1980 = 2.19; \\ \mathsf{EP} &= 2.19 - 10 = -7.81. \end{aligned}$$