Bayesian Networks and Influence Diagrams: A Guide to Construction and Analysis
Answers to Chapter 5

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Answers to Exercises

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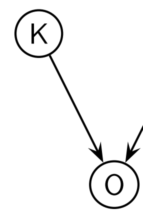
Answer to Exercise 5.1:

(a) The structure of a Bayesian network representation is shown below where S is the initial selection, K is the location of the price, and O is the door opened by the official.



(b) The probability distribution is $P(K) = (1/3, 0, 2/3)$

Answer to Exercise 5.2:

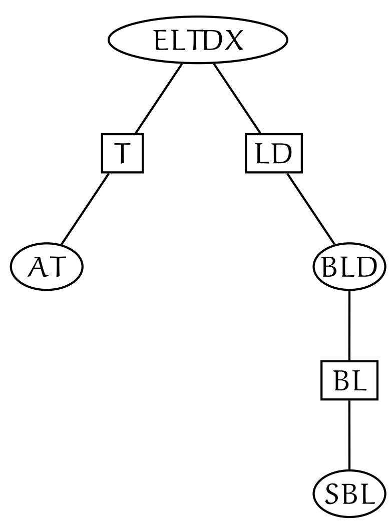
(a) Q_1: {Asia, Tuberculosis, Tub_or_cancer, X_ray, Cancer, Dysponea},
Q_2: {X_ray},
Q_3: $\emptyset$.

(b) Q_1: {Smoker},
Q_2: {Asia, Tuberculosis},
Q_3: {Asia, Tuberculosis}.

Answer to Exercise 5.3:

(a) {Tub_or_cancer, Cancer, Tuberculosis, Dysponea, X_ray},
{Asia, Tuberculosis},
{Smoker, Bronchitis, Cancer},
{Bronchitis, Cancer, Dysponea},
{Cancer, Tuberculosis, Dysponea, X_ray},
{Tuberculosis, Dysponea, X_ray},
{Tuberculosis, Dysponea}, and {Tuberculosis},

(b) A junction tree representation is shown below:



(c) The largest clique in the junction tree representation in (b) has five members whereas the largest clique in the junction tree representation on page 117.

Answer to Exercise 5.4:

(a) The set of relavant nodes for each decision specified in the table below:

Decision node	Relevant nodes
D_1	$\{A, C, D, E, F, G, H, I, J, K, L, D_2, D_3, D_4\}$
D_2	$\{G, I, L, D_4\}$
D_3	$\{H, J, K\}$
D_4	$\{I, L\}$

(b) The set of requisite nodes for each decision specified in the table below:

Decision node	Requisite nodes
D_1	$\{B\}$
D_2	$\{E\}$
D_3	$\{F\}$
D_4	$\{G, D_2\}$

(c) The total order on the decision nodes induce the following partial order on the chance nodes relative to the decision nodes:

$$\{B\} \prec D_1 \prec \{E, F\} \prec D_2 \prec \emptyset \prec D_3 \prec \{G\} \prec D_4 \prec \{A, C, D, H, I, J, K, L\}$$

(d) The domain of the decision policy for each decision node consists, in principle, of the decision node and all the nodes preceeding the decision in the order identified in (c). Each decision depends, however, only on the requisite nodes (as identified in (b)).

Answer to Exercise 5.5:

(a) $\text{dom}(\delta_{D_1}) = \{B, D_1\}$, $\text{dom}(\delta_{D_2}) = \{E, D_2\}$, $\text{dom}(\delta_{D_3}) = \{D_3\}$ (D_2 can be identified as non-requisite for D_3), and $\text{dom}(\delta_{D_4}) = \{D_4\}$ (D_3 can be identified as non-requisite for D_4).

(b) None of the decisions are extreme.

Answer to Exercise 5.6:

(a) Since decision D_1 (first choice of door) is taken without any information available and the value of variable O (officials choice) is known before we take the final decision, D_2, we have

$$I_0 = \{\}, I_1 = \{O\}, I_2 = \{K\},$$

where K denotes the correct door. This gives us

$$\mathrm{MEU}(D_1) = \max_{D_1} \sum_{O} \max_{D_2} \sum_{K} P(O \mid D_1, K) P(K) U(K, D_2),$$

as the officials choice depends on my first choice (D_1) and on where the money is (K), and as the utility, U, is a function of where the money is (K) and of my final choice (D_2). We have

$P(O \mid D_1, K)$	A	B	C
A	$(0, 0.5, 0.5)$	$(0, 0, 1)$	$(0, 1, 0)$
B	$(0, 0, 1)$	$(0.5, 0, 0.5)$	$(1, 0, 0)$
C	$(0, 1, 0)$	$(1, 0, 0)$	$(0.5, 0.5, 0)$

$U(D2, K)$	A	B	C
A	30000	0	0
B	0	30000	0
C	0	0	30000

and $P(K)$ is the uniform distribution. The product $P(O \mid D_1, K)P(K)U(K, D_2)$ is given in the table below, where the triples range over the states of K.

D_1	A	A	A	B	B	B	C	C	C
D_2	A	B	C	A	B	C	A	B	C
A	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 5000, 0)$	$(0, 0, 10000)$	$(0, 0, 0)$	$(0, 10000, 0)$	$(0, 0, 5000)$
B	$(5000, 0, 0)$	$(0, 0, 0)$	$(0, 0, 10000)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(10000, 0, 0)$	$(0, 0, 0)$	$(0, 0, 5000)$
C	$(5000, 0, 0)$	$(0, 10000, 0)$	$(0, 0, 0)$	$(10000, 0, 0)$	$(0, 5000, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$

Marginalizing out K from this tables gives us the following table.

D_1	A	A	A	B	B	B	C	C	C
D_2	A	B	C	A	B	C	A	B	C
A	0	0	0	0	5000	10000	0	10000	5000
B	5000	0	10000	0	0	0	10000	0	5000
C	5000	10000	0	10000	5000	0	0	0	0

Next, we eliminate D_2 through max-marginalization, which gives us the following table:

D_1	A	B	C
A	0	10000	10000
B	10000	0	10000
C	10000	10000	0

Finally, we sum out O and get

D_1	A	B	C
	20000	20000	20000

As the expected utilities are identical, the answer is

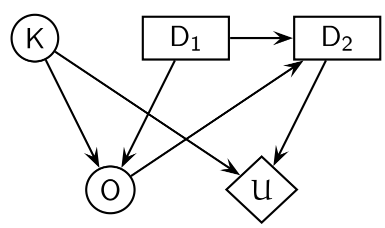
$$\text{MEU}(D_1) = 20000.$$

(b) Given that you have chosen door A and given knowledge about which door is the correct one, the decision scenarios for the official are as follows:

Secret knowledge	Official's options
A is correct	B or C
B is correct	C
C is correct	B

Thus, in two of the three, equally likely, cases (i.e., where you have selected doors with nothing behind), the choice of the official gives you certain information about the location of the money. The information provided to you in the third case (i.e., where you have selected the right door) does not tell you anything about the correctness of your choice. In conclusion, in two out of three cases it pays to alter your choice, meaning that in average it is twice as likely that the money hides behind the door not selected by either you or the official.

(c) The influence diagram corresponding to the problem appears below.



When implemented and run in Hugin (without evidence), we get a maximum expected utility for D_1 of \$20,000. By entering evidence corresponding to, for example, door A as our first choice and door B as the officials choice, we find probabilities consistent with those calculated above. Also we note that the probability distributions for K are proportional to the expected utilities for your second decision alternatives.

(d) The optimal policy is to change choice.