Bayesian Networks and Influence Diagrams: A Guide to Construction and Analysis

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Answers to Exercises

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Answer to Exercise 6.1:

(a) Focusing on the problem (i.e., who will be pardoned?) and the available information (i.e., the information from the warden), we can arrive at a problem variable, say Pardoned (or P for short), and an information variable, say Warden (or W for short).

(b) $\mathrm{dom}(\mathsf{P}) = \{\mathsf{A}, \mathsf{B}, \mathsf{C}\}$, $\mathrm{dom}(W) = \{\mathsf{B}, \mathsf{C}\}$.

(c) $\mathrm{dom}(\mathsf{P})$ and $\mathrm{dom}(W)$ are clearly exhaustive and each of them consist of mutually exclusive states. Also, the states of P and W are not mutually exclusive across $\mathrm{dom}(\mathsf{P})$ and $\mathrm{dom}(W)$, meaning that P and W pass the uniqueness test. Hence, P and W are well-defined.

(d) P is a problem variable and W is an information variable providing symptom information. Hence, P and W are linked through a directed link from P to W.

(e)

$$P(\mathsf{P}) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) \qquad \text{and} \qquad P(W \mid \mathsf{P}) = $$

	P = A	P = B	P = C
W = B	$\frac{1}{2}$	0	1
W = C	$\frac{1}{2}$	1	0

Given $W = \mathsf{B}$ and since $P(W = \mathsf{B} \mid \mathsf{P} = \mathsf{B}) = 0$, we find that

$$\begin{aligned} P(\mathsf{P} = \mathsf{A} \mid W = \mathsf{B}) &= \frac{P(\mathsf{P} = \mathsf{A})P(W = \mathsf{B} \mid \mathsf{P} = \mathsf{A})}{P(\mathsf{P} = \mathsf{A})P(W = \mathsf{B} \mid \mathsf{P} = \mathsf{A}) + P(\mathsf{P} = \mathsf{C})P(W = \mathsf{B} \mid \mathsf{P} = \mathsf{C})} \\ &= \frac{\frac{1}{3} \cdot \frac{1}{2}}{\frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot 1} \\ &= \frac{1}{3} \end{aligned}$$

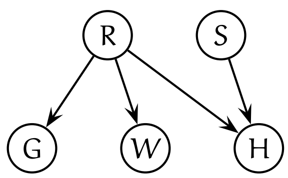
Hence, A was wrong, as his chances are still $\frac{1}{3}$. The chances of prisoner C, however, goes up to $\frac{2}{3}$. (Some might be puzzled about this result, so let us try to convey an intuitive explanation: The chances that one of prisoners B or C is going to be pardoned is of course $\frac{2}{3}$. So a probability of $\frac{2}{3}$ is attributed to the pair (B, C) and $\frac{1}{3}$ to A. Now, getting the information that prisoner B (or C) is not going to be pardoned should not make us change our belief about the chances of prisoner A being pardoned, as the information does not tell us anything new about the chances that one of the prisoners of the pair (B, C) will be pardoned. We knew from the beginning that one of them would be executed, and getting to know that prisoner B (or C) will be excuted only makes us move the entire probability mass for the pair (B, C) to C (or B).)

Answer to Exercise 6.2: If we were to define one variable for each prisoner, the domain of these would be, say, {pardon, execute}. This constitutes an exhaustive set of mutually exclusive states as it should. But some states are mutually exclusive across the variables as one and only one of the variables must be in state pardon. Therefore, the variables do not pass the uniqueness test, and are hence not well-defined.

Answer to Exercise 6.3:

(a)-(b) Thinking in terms of the problem (i.e., what might be the cause of wet grass) and the available information (i.e., observations of the states of the lawns), we identify two problem variables, say Rain (R) and Sprinkler (S), with domains {no, yes} and {off, on}, respectively, and three symptom variables, say HolmesLawn (H), WatsonsLawn (W), and GibbonsLawn (G), all with domains {dry, wet}.

(c) As rain and sprinkler both can cause the lawns to get wet, the relations between the two problem variables and the three symptom variables are of cause-consequence nature, hence the cause-consequence idiom would be the natural choice. We then get the following structure



(d) Using the d-separation criterion on the above structure gives us that

(i) $H \perp\!\!\!\perp W \mid R$, $H \perp\!\!\!\perp G \mid R$, $W \perp\!\!\!\perp G \mid R$, $H \not\perp\!\!\!\perp W$, $H \not\perp\!\!\!\perp G$, $W \not\perp\!\!\!\perp G$.

(ii) $R \not\perp\!\!\!\perp S \mid H$, $R \perp\!\!\!\perp S$, $R \perp\!\!\!\perp S \mid W$, $R \perp\!\!\!\perp S \mid G$.

Answer to Exercise 6.4:

(a) B and E are problem variables, R and W are symptom variables, and A is both a symptom variable (relative to B and E) and a problem variable (relative to W), connecting two causal links ($B \rightarrow A$ and $A \rightarrow W$) forming a causal chain $B \rightarrow A \rightarrow W$.

(b) The structure is consistent with the prototypical causal structure as there are links from problem variables B and E to symptom variables A and R, and there is a link from symptom/problem variable A to symptom variable W.